



**Coimisiún na Scrúduithe Stáit
State Examinations Commission**

LEAVING CERTIFICATE EXAMINATION, 2006

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 23 JUNE – AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of g to be 10 m/s^2 .

\vec{i} and \vec{j} are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Marks may be lost if necessary work is not clearly shown.

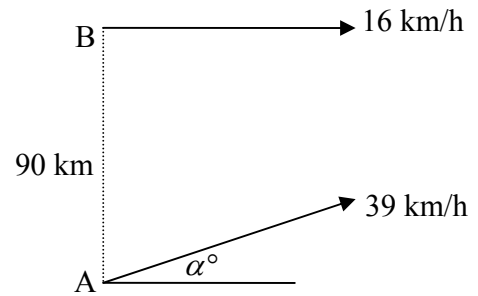
1. A car travels along a straight level road.
 It passes a point p at a speed of 10 m/s and accelerates uniformly for 5 seconds to a speed of 30 m/s.
 It then moves at a constant speed of 30 m/s for 9 seconds.
 Finally the car decelerates uniformly from 30 m/s to rest at point q in 6 seconds.

- Find
- (i) the acceleration
 - (ii) the deceleration
 - (iii) $|pq|$, the distance from p to q
 - (iv) the average speed of the car as it travels from p to q .

2. Ship A is travelling east α° north with a constant speed of 39 km/h, where $\tan \alpha = \frac{5}{12}$.

Ship B is travelling due east with a constant speed of 16 km/h.

At 2 pm ship B is positioned 90 km due north of ship A.

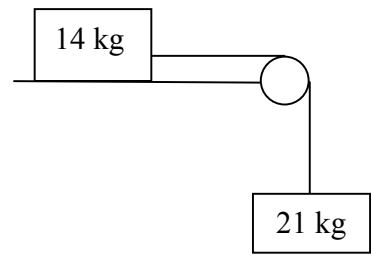


- (i) Express the velocity of ship A and the velocity of ship B in terms of \vec{i} and \vec{j} .
- (ii) Find the velocity of ship A relative to ship B in terms of \vec{i} and \vec{j} .
- (iii) Find the shortest distance between the ships.

3. A particle is projected from a point on a level horizontal plane with initial velocity $10\vec{i} + 35\vec{j}$ m/s, where \vec{i} and \vec{j} are unit perpendicular vectors in the horizontal and vertical directions respectively.

- Find
- (i) the time it takes to reach the maximum height
 - (ii) the maximum height
 - (iii) the two times when the particle is at a height of 50 m
 - (iv) the speed with which the particle strikes the plane.

4. (a) Two particles of masses 14 kg and 21 kg are connected by a light, taut, inextensible string passing over a smooth light pulley at the edge of a rough horizontal table. The coefficient of friction between the 14 kg mass and the table is $\frac{1}{2}$.

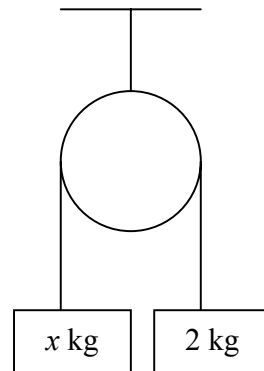


The system is released from rest.

- (i) Show on separate diagrams the forces acting on each particle.
 (ii) Find the common acceleration of the particles.

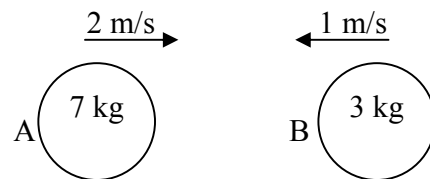
- (b) A light inelastic string passes over a smooth light pulley. A mass of x kg is attached to one end of the string and a mass of 2 kg is attached to the other end.

When the system is released from rest the 2 kg mass falls 3 metres in $\sqrt{6}$ seconds.



- Find (i) the common acceleration
 (ii) the tension in the string
 (iii) the value of x .

5. A smooth sphere A, of mass 7 kg, collides directly with another smooth sphere B, of mass 3 kg, on a smooth horizontal table. A and B are moving in opposite directions with speeds of 2 m/s and 1 m/s respectively.



The coefficient of restitution for the collision is $\frac{1}{3}$.

- Find (i) the speed of A and the speed of B after the collision
 (ii) the loss in kinetic energy due to the collision
 (iii) the magnitude of the impulse imparted to A due to the collision.

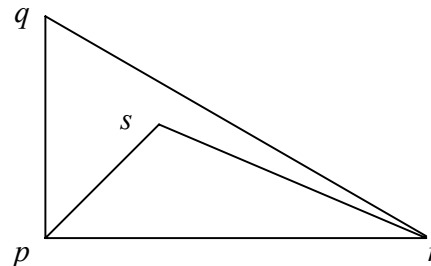
6. (a) Particles of weight 3 N, 7 N, 10 N and 15 N are placed at the points $(-4, -5)$, $(2, 1)$, (x, y) and $(-1, 3)$, respectively.
The centre of gravity of the four particles is at the origin.

Find the value of x and the value of y .

- (b) A triangular lamina with vertices p , q and r has the triangular portion with vertices p , s and r removed.

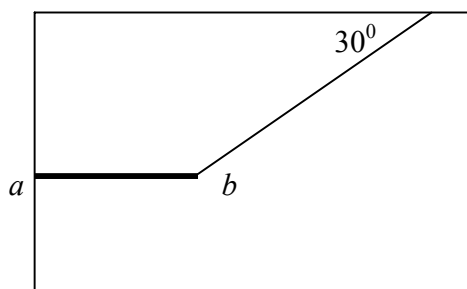
The co-ordinates of the vertices are $p(0, 0)$, $q(0, 6)$, $r(12, 0)$ and $s(3, 3)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.



7. A uniform rod, ab , of length 4 m and weight 80 N is smoothly hinged at end a to a vertical wall. One end of a light inelastic string is attached to b and the other end of the string is attached to a horizontal ceiling.

The string makes an angle of 30° with the ceiling, as shown in the diagram.



The rod lies horizontally and in equilibrium.

- (i) Show on a diagram all the forces acting on the rod ab .
- (ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
- (iii) Write down the equation that arises from taking moments about point a .
- (iv) Find the tension in the string.
- (v) Find the magnitude and direction of the reaction at the hinge.

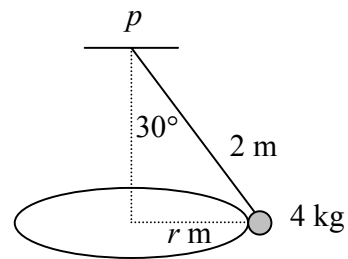
8. (a) A particle describes a horizontal circle of radius 2 metres with constant angular velocity ω radians per second.

The particle completes one revolution every 5 seconds.

- (i) Show that ω is equal to $\frac{2\pi}{5}$.
- (ii) Find the speed and acceleration of the particle.
Give your answers correct to one place of decimals.

- (b) A conical pendulum consists of a particle of mass 4 kg attached by a light inelastic string of length 2 metres to a fixed point p .

The particle describes a horizontal circle of radius r . The centre of the circle is vertically below p .



The string makes an angle of 30° with the vertical.

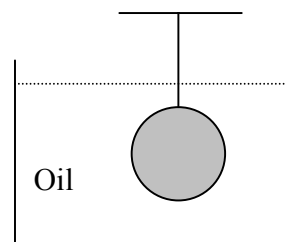
- Find
- (i) the value of r
 - (ii) the tension in the string
 - (iii) the speed of the particle.

9. (a) State the Principle of Archimedes.

A solid piece of metal weighs 150 N in air and 131 N in water.
Find the volume of the piece of metal.

- (b) A solid sphere of radius 5 cm and relative density 8 is completely immersed in oil of relative density 0.9.

The sphere is held at rest by a light inelastic vertical string which is tied to a fixed support.



Find the tension in the string.

[Density of water = 1000 kg/m^3].

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