# Coimisiún na Scrúduithe Stáit
# State Examinations Commission

## LEAVING CERTIFICATE EXAMINATION, 2014

## APPLIED MATHEMATICS – ORDINARY LEVEL

**FRIDAY, 20 JUNE – MORNING, 9.30 to 12.00**

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of $g$ to be 10 m $s^{-2}$.

$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

**Marks may be lost if necessary work is not clearly shown.**

**1.** The points $P$ and $Q$ lie on a straight level road.
A car passes point $P$ with a constant speed of 13 m $s^{-1}$ and continues at this speed for 9 seconds.
The car then accelerates uniformly for 5 seconds to a speed of 28 m $s^{-1}$.
Finally the car decelerates uniformly from 28 m $s^{-1}$ to rest at point $Q$.
The car travels 98 metres while decelerating.

**(a)** Draw a speed-time graph of the motion of the car from $P$ to $Q$.

**(b)** Find **(i)** the acceleration

**(ii)** the deceleration

**(iii)** $|PQ|$, the distance from $P$ to $Q$

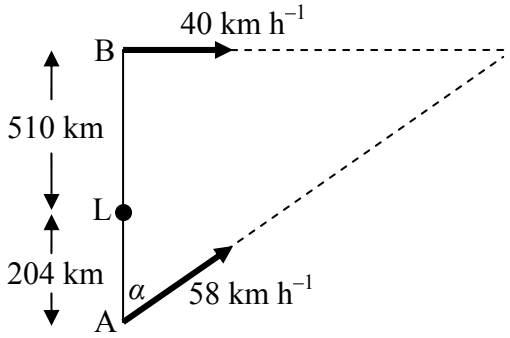
**(iv)** the average speed of the car as it travels from $P$ to $Q$, correct to two decimal places.

**2.** Ship A is positioned 204 km due south of lighthouse L.

A is moving at an angle $\alpha$ east of north at a constant speed of 58 km $h^{-1}$, where $\tan \alpha = \frac{20}{21}$.

Ship B is positioned 510 km due north of lighthouse L.

B is moving due east at a constant speed of 40 km $h^{-1}$.




Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

**(ii)** the velocity of B in terms of $\vec{i}$ and $\vec{j}$

**(iii)** the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$.

Ship A intercepts ship B after $t$ hours.

Find **(iv)** the value of $t$

**(v)** the distance from lighthouse L to the meeting point.

**3.** A particle is projected from a point on horizontal ground with an initial speed of 82 m $s^{-1}$ at an angle $\beta$ to the horizontal, where $\tan\beta = \frac{40}{9}$.

Find **(i)** the initial velocity of the particle in terms of $\vec{i}$ and $\vec{j}$

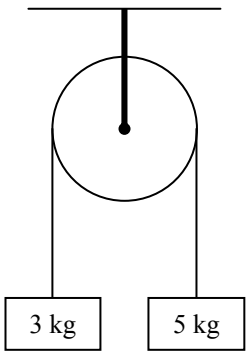
**(ii)** the time taken to reach the maximum height

**(iii)** the maximum height of the particle above ground level

**(iv)** the range

**(v)** the two times at which the height of the particle is 275 m.

**4.** **(a)** Two particles of masses 3 kg and 5 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley.

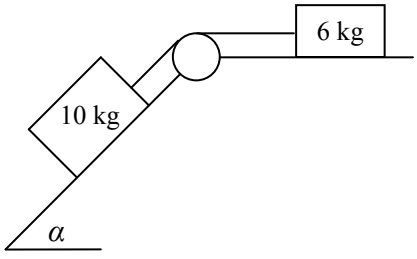



The system is released from rest.

Find **(i)** the common acceleration of the particles

**(ii)** the tension in the string.

**(b)** Masses of 6 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light fixed pulley as shown in the diagram.




The 6 kg mass lies on a rough horizontal plane and the coefficient of friction between the 6 kg mass and the plane is $\frac{2}{3}$.
The 10 kg mass lies on a smooth plane which is inclined at an angle $\alpha$ to the horizontal, where $\tan\alpha = \frac{4}{3}$.

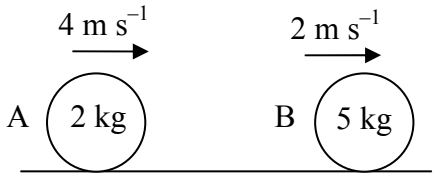
The system is released from rest.

**(i)** Show on separate diagrams the forces acting on each particle.

**(ii)** Find the common acceleration of the masses.

**(iii)** Find the tension in the string.

**5.** A smooth sphere A, of mass 2 kg, collides directly with another smooth sphere B, of mass 5 kg, on a smooth horizontal table.




A and B are moving in the same direction with speeds of 4 m s$^{-1}$ and 2 m s$^{-1}$ respectively.

The impulse imparted to B due to the collision is 5 N s.

Find **(i)** the speed of B after the collision

**(ii)** the speed of A after the collision

**(iii)** the coefficient of restitution for the collision

**(iv)** the loss in kinetic energy due to the collision.

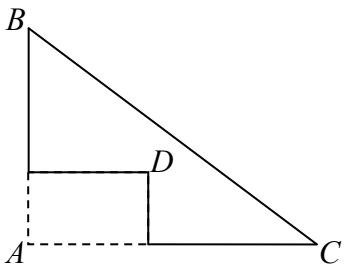
**6.** **(a)** Particles of weight 8 N, 2 N, 7 N and 3 N are placed at the points $(6, p)$, $(-4, q)$, $(p, 4)$ and $(11, 6)$ respectively.

The co-ordinates of the centre of gravity of the system are $(4, q)$.

Find **(i)** the value of $p$

**(ii)** the value of $q$.

**(b)** A triangular lamina with vertices $A$, $B$ and $C$ has the rectangle with diagonal $[AD]$ removed.

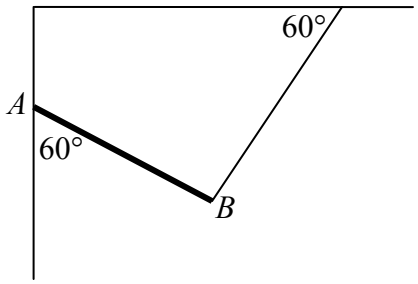



The co-ordinates of the points are $A(0, 0)$, $B(0, 18)$, $C(24, 0)$ and $D(10, 6)$.

Find the co-ordinates of the centre of gravity of the remaining lamina.

**7.** A uniform rod, [$AB$], of length 2 m and weight 120 N is smoothly hinged at end $A$ to a vertical wall.

One end of a light inelastic string is attached to $B$ and the other end of the string is attached to a horizontal ceiling.




The string makes an angle of 60° with the ceiling and the rod makes an angle of 60° with the wall, as shown in the diagram.

The rod is in equilibrium.

**(i)** Show on a diagram all the forces acting on the rod [$AB$].

**(ii)** Write down the two equations that arise from resolving the forces horizontally and vertically.

**(iii)** Write down the equation that arises from taking moments about the point $A$.

**(iv)** Find the tension in the string.

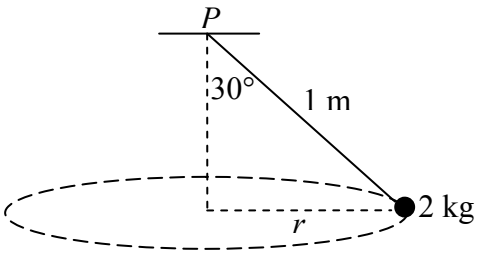
**(v)** Find the magnitude of the reaction at the point $A$.

**8.** **(a)** A particle describes a horizontal circle of radius 2 metres with uniform angular velocity $\omega$ radians per second.
The period $T$ (the time to travel one complete circle) is $0.4\pi$ seconds.

Find **(i)** the value of $\omega$
**(ii)** the speed of the particle
**(iii)** the acceleration of the particle.

**(b)** A conical pendulum consists of a particle of mass 2 kg attached by a light inelastic string of length 1 metre to a fixed point $P$.
The string makes an angle of 30° with the vertical.

The particle describes a horizontal circle of radius $r$ and the centre of the circle is vertically below $P$.



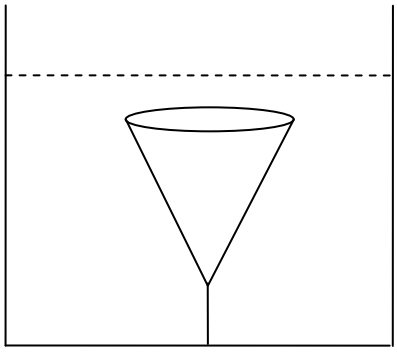

Find **(i)** the value of $r$
**(ii)** the tension in the string
**(iii)** the angular velocity of the particle.

**9.** **(a)** State the principle of Archimedes.

A solid piece of metal has a weight of 35 N.
When it is completely immersed in water, the metal appears to weigh 27 N.

Find **(i)** the volume of the metal
**(ii)** the density of the metal.

**(b)** A right circular solid cone has a base of radius 4 cm and a height of 12 cm.



The relative density of the cone is 0.9 and it is completely immersed in a tank of liquid of relative density 1.3.

The cone is held at rest by a light, inextensible, vertical string which is attached to the base of the tank. The upper surface of the cone is horizontal.

Find the tension in the string.

[Density of water = 1000 kg $m^{-3}$]

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